Comments on Two-Phase Measurements Using a Resistive Probe

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A timely contribution to two-phase measurement techniques is described by Bankoff and Neal (*1*). Their resistive-probe technique for measuring local void fractions should prove valuable to those who desire to know more about local properties in two-phase flow. In reference *1*, the authors show expressions for the volumetric fraction of the gas, the median chord of a bubble pierced by the probe, and size distributions of gas and liquid. Their development treats the expressions separately. For any bubbles, the median chord is independent of motion, since it is obtained from the one-half total volume of a bubble. The average chord represents the ratio of total volume to area normal to the flow, and hence depends on fluid motion. The dependency links it to volumetric fraction and size distributions.

Using the concept of average bubble chord, these comments present the development of relationships between local time-averaged and space-averaged properties and illustrate the difficulties of interpreting in terms of bubble size the probe time traces caused by spherical gas bubbles. Possibilities of future work in local measurements are discussed.

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Bubble Radius Distribution Functions from Resistivity Probe Measurements

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Anderson (*1*) derives a formula relating the bubble radius probability density function (*2*) to the contact-time density function on the assumption that the bubble-rise velocity is independent of position; that is, $U = U(\zeta)$. It is probably more realistic in most systems to assume that $U = U(\zeta, s)$. Here a formula for this situation is derived by a method somewhat different than the one employed by Anderson.

Assume that the probe is positioned at a dimensionless radial position, s_0. Also, with respect to polar coordinates fixed at the center of the bubble, the z axis pointing upward and the x-axis pointing toward the center of the pipe, the position of the probe tip at the instant of contact with the bubble is (ζ, θ, ϕ). The triangle cosine law then requires that

$$s_0^2 = \zeta^2 \sin^2 \theta + s^2 - 2s\zeta \sin\theta \cos\phi \qquad (1)$$

Now the joint probability density function of the simultaneous event consisting of a particular combination of the random variables ζ, θ, and ϕ is given by

$$f(\zeta, \theta, \phi) = [b(\zeta)] [2 \cos \theta \sin \theta] \left[\frac{1}{2\pi}\right] \qquad (2)$$

where it is assumed that the three events are unrelated, so that their joint probability is the product of the individual probability densities; that any element of the projected bubble frontal area has the same probability of being struck as any other element; and that the azimuthal angle probability density is a constant.

Consider first the case where the bubble-rise velocity depends only on bubble radius; that is, $U = U(\zeta)$. It is convenient to transform the joint probability density function to a new set of random variables (ζ, τ, ϕ) by evaluating the Jacobian $J_1 = \dfrac{\partial(\zeta, \theta, \phi)}{\partial(\zeta, \tau, \phi)}$ where the contact time is given by

$$\tau = \frac{2 R \zeta \cos\theta}{U} \qquad (3)$$

assuming that the bubble is not deflected or slowed down by the probe. In this way it is found that

$$J_1 = \frac{\partial\theta}{\partial\tau} = -\frac{U}{2 R \zeta \sin\theta} \qquad (4)$$

which leads to

$$f(\zeta, \tau, \phi) = \frac{b(\zeta)\tau}{\pi}\left(\frac{U}{2\zeta R}\right)^2 \qquad (5)$$

The probability density function, $p(\tau)$, for bubble contact times is a marginal distribution function (3) obtained by integration over two dimensions:

$$p(\tau) = 2\tau \int_{\zeta_{\min}}^{\zeta_{\max}} \left(\frac{U}{2R\zeta}\right)^2 b(\zeta)\, d\zeta \qquad (6)$$

This is the same as Anderson's Equation (2). However, the bubble-rise

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INFORMATION RETRIEVAL

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Countercurrent heat exchange with a vaporizing immiscible transfer agent, Harriott, Peter, and Herbert Wiegandt, **A.I.Ch.E. Journal, 10,** No. 5, p. 755 (September, 1964).

Key Words: Mass Transfer-8, Heat Transfer-8, Packed Beds-10, Countercurrent-10, Evaporation-8, Condensation-8, Water-4.

Abstract: Countercurrent, staged, heat transfer between two aqueous streams was carried out by vaporizing an immiscible transfer agent from the stream being cooled and condensing it in the stream being warmed. The coefficient of heat transfer in the packed condenser was approximately 150,000 B.t.u./hr.cu.ft. °F. at a liquid rate of 40,000 lb./sq.ft.hr. Comparable performance was obtained from the evaporator when there was present more than 6 vol. % of the transfer agent.

Mass transfer in a packed, pulsed column, Krasuk, J. H., and J. M. Smith, **A.I.Ch.E. Journal, 10,** No. 5, p. 759 (September, 1964).

Key Words: Mass Transfer-8, Pulsation-10, 4, Packed Column-5, 10, Frequency-6, Amplitude-6, Beta Naphthol-1, Water-5, Solution-9, Particle size-6, Porosity-7.

Abstract: Equations in terms of a pulse Reynolds number and dimensionless pulse velocity are presented for mass transfer rates in pulsed flow in a packed bed. Mass transfer from the wall and from the particles is considered.

The experimental measurements consisted of rates of dissolution of β-naphthol from the wall and from the particles within the bed. The improvement in $\bar{k}_p$ increased with frequency and decreased with average flow rate, although the increase was always less than in an empty tube at equivalent conditions.

Generalized correlation of the constants of the Benedict-Webb-Rubin-Friend equation for paraffinic isomers, Beyer, Harold H., and Richard G. Griskey, **A.I.Ch.E. Journal, 10,** No. 5, p. 764 (September, 1964).

Key Words: Generalized Correlation-8, Benedict-Webb-Rubin-Friend Equation of State-8, Isomeric Paraffins-9, 3-Methyl Paraffins-9, 2,2-Dimethyl Paraffins-9, 2,3-Dimethyl Paraffins-9, Pressures-2, Superheated Vapor Region-.

Abstract: A generalized correlation of the constants of the Benedict-Webb-Rubin-Friend equation of state was developed for isomeric paraffins. The correlation was applied to 3-methyl, 2,2-dimethyl, and 2,3-dimethyl paraffins.

Pressures calculated in the superheated vapor region deviated by less than 1% from experimental data. Critical pressure values were also precisely reproduced not only for the compounds whose constants were extrapolated.

A method is outlined for extending the correlation to other isomeric paraffins when little experimental data is available.

Thermodynamics and interfacial tension of multicomponent liquid-liquid interfaces, Shain, Stephen A., and J. M. Prausnitz, **A.I.Ch.E. Journal, 10,** No. 5, p. 766 (September, 1964).

Key Words: A. Interfacial Tension-9, 2, Concentration-6, Molecular Energy Parameter-6, 1, Molecular Size-6, 1, Activity Coefficients-6, 1, Thermodynamics-10. B. Interfacial Tension-7, Concentration-6, Capillary-10, Alcohols-4, Water-5, Organic Solvent-5.

Abstract: Surface thermodynamics and solution theory are combined to calculate the derivative of interfacial tension with respect to solute concentration. An equation is derived for ternary systems relating the interfacial tension to molecular properties of the solvent and solute species. Experimental data are reported for nine ternary systems consisting of an aqueous phase and an organic phase where both phases are dilute solutions of an alcohol.

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velocity depends upon the radial position, as well as the radius, of the bubble; that is, $U = U(\zeta, s)$. Now the probability density function $f(\zeta, \theta, \phi)$ may be transformed to a new set of coordinates by means of the Jacobian, $J_2 = \dfrac{\partial(\zeta, \theta, \phi)}{\partial(\zeta, \tau, s)}$. After a little manipulation it is found that

$$J_2 = \frac{\partial\theta}{\partial\tau}\bigg|_{s,\zeta} \frac{\partial\phi}{\partial s}\bigg|_{\zeta,\theta} = \left(\frac{-U}{2\,\zeta\,R\sin\theta}\right) \left(\frac{\cot\phi}{s} - \frac{\csc\theta\csc\phi}{\zeta}\right) \quad (7)$$

which leads to

$$p(\tau) = \frac{1}{2\pi R}\bigg| \int_{\zeta_{\min}}^{\zeta_{\max}} \int_{s_o-\zeta}^{s_o+\zeta} \frac{b(\zeta)\,U(\zeta, s)}{\zeta}\left(\frac{\cot\phi}{s} - \frac{\csc\theta\csc\phi}{\zeta}\right) \cos\theta\,ds\,d\zeta \bigg| \quad (8)$$

where ϕ and θ are obtained from Equations (1) and (3). The bubble-rise velocity is given, to a good degree of approximation, by the sum of the stagnant rise velocity and the local liquid velocity:

$$U(\zeta, s) = U_b(\zeta) + U_L(s) \quad (9)$$

Probably the most convenient way to solve the integral Equation (8) is to approximate $b(\zeta)$ by some suitable trial function with two or more constants which are determined by making the integral on the right-hand side conform with minimum square error to the experimental data for $p(\tau)$.

NOTATION

$b(\zeta)$ = bubble radius probability density function
J_1, J_2 = Jacobian determinants
$p(\tau)$ = contact-time density function
R = pipe radius
s = radial position of the bubble center, dimensionless
s_o = radial position of the probe tip, dimensionless
U = bubble rise velocity

Greek Letters

θ = spherical polar angle
ϕ = azimuthal angle
τ = author, pls supply definition
ζ = z/R, dimensionless axial position

LITERATURE CITED

1. Anderson, T. T., *A.I.Ch.E. Journal,* **10,** No. 5, p. 776 (1964).
2. Neal, L. G., and S. G. Bankoff, *A.I.Ch.E. Journal,* **9,** 490 (1963).
3. Condon, E. U., and H. P. Odishaw, ed., "Handbook of Physics," Sect. 1, p. 139, McGraw-Hill, New York (1958).